# MATH 2B Review: Chain Rule

## Facts to Know:

Derivative formula for composition of functions.

Chain Rule 1: $\frac{d}{dx} f(g(x)) =$ $f'\left(g(x)\right) g'(x)$

$\uparrow$ chain

Chain Rule 2: $y = f(u)$ and $u = g(x)$, $\frac{dy}{dx} =$ $\dfrac{dy}{du} \cdot \dfrac{du}{dx}$

## Examples:

1. Find the derivative of $y = e^{6x}$.  $= f(g(x))$

$f(x) = e^x$

$g(x) = 6x$

$f'(x) = e^x$

$g'(x) = 6$

$\dfrac{dy}{dx} = f'(g(x)) g'(x) = e^{6x} \cdot 6 = \boxed{6e^{6x}}$

$y = f(u) = e^u$

$u = g(x) = 6x$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx} = e^u \cdot 6 = 6e^u = \boxed{6e^{6x}}$

2. Find the derivative of $y = \sin^3(2x)$.  $= \left(\sin(2x)\right)^3 = f(g(h(x)))$

$f(x) = x^3 \qquad g(x) = \sin(x) \qquad h(x) = 2x$

$f'(x) = 3x^2 \qquad g'(x) = \cos(x) \qquad h'(x) = 2$

$\dfrac{d}{dx} f(g(h(x))) = f'(g(h(x))) \dfrac{d}{dx} g(h(x))$

$= f'(g(h(x))) g'(h(x)) h'(x)$

$= 3\left(\sin(2x)\right)^2 \cdot \cos(2x) \cdot 2$

$= \boxed{6\cos(2x) \sin^2(2x)}$